Magnet Field Quality and Lattice Design Options

S. Peggs
November 16, 1998

1. Introduction
2. Tune Shifts
3. Maximum Systematic Errors
4. Summary and Conclusions

Introduction

This presentation has 2 goals:

1. Make semi-quantitative estimates of *tolerable systematic harmonics* in arc dipoles in a high or low field Future Hadron Collider.
2. Draw the connection between field quality b_n and maximum (optimum) half cell length L.

• Conventional post-SSC wisdom is that **systematic errors dominate randoms** in (low temperature) superconducting machines.

• Accelerator Physics analysis is then simplified, emphasizing **tune shifts, not resonances**, and making general scaling rules possible.

• ... but note that Talman, Verdier & others are still working to "suppress" resonances at LHC by optimizing integer tune splits.

Passing comments:

• Geometric systematics can be reduced during early industrial production.

• Such reduction in RHIC means that the octupole & decapole correctors in the spool pieces will not be powered. (Landau damping?)

• Reducing the number and complexity of spool pieces is a radical route to significant cost savings. Ends cost. Long cells save.

• Spool money may be saved by maximizing the half cell length L, beyond the conventional 53.4 meters of the LHC, and 100 meters of the SSC.

• Can tuning shims be used to reduce several harmonics at injection in each arc dipole, *after* it has been measured warm?

<u>Tune Shifts</u>

Apologies for 3 pages of AP formulae!

$$B_y \quad = \quad B_0 \left[1 \; + \; \sum_n \frac{b_n}{r_0^n} \, x_t^n \right] \tag{1}$$

• The tune shift depends on the **betatron amplitude** and the (constant) **momentum offset**, parameterized by m_x and m_δ

$$A_x \quad = \quad m_x \, \sigma_x \tag{2}$$

$$\eta\delta \quad = \quad m_\delta \, \sigma_\delta \tag{3}$$

RMS betatron & mmtm. beam sizes are σ_x & σ_δ.

• **Assumption 1:** the RF system (et cetera) is manipulated so that

$$\widehat{\sigma_\delta} \; = \; \widehat{\sigma_x} \tag{4}$$

at F quads. Not unreasonable in practice.

• **Assumption 2:** the arcs are made from a standard FODO cells with a phase advance of ϕ_c.

• Then

$$\Delta Q_x = \frac{b_n}{r_0^n(1+\delta)} L^{(n+1)/2} \left(\frac{\epsilon_x}{\beta\gamma}\right)^{(n-1)/2}$$

$$\times \sum_{i=0}^{n-1-2i\geq 0} C_{n,i}\alpha_{n,i}(\phi_c)\, m_\delta^{n-1-2i}\, m_x^{2i}$$

where ϵ_x is the RMS emittance.

• **The first piece of this expression already shows how b_n, L, ϵ_x, and $(\beta\gamma)$ may be traded off!**

• The sum contains messy coefficients $C_{n,i}$ and $\alpha_{n,i}(\phi_c)$. For example

$$C_{n,i} = \frac{1}{2^{2i+1}} \frac{n!(2i+2)!}{(n-2i-1)!(2i+1)!(i+1)!(i+1)!} \tag{5}$$

- The first few $C_{n,i}$ values are:

n	Multipole	$i = 0$	1	2
1	Quadrupole	1/2		
2	Sextupole	1		
3	Octupole	3/2	3/8	
4	Decapole	2	3/2	
5	12-pole	5/2	15/4	5/16
6	14-pole	3	15/2	15/8

- If $\phi_c = 90$ degrees, the first few $\alpha_{n,i}$ values are:

n	Multipole	$i = 0$	1	2
1	Quadrupole	1.667		
2	Sextupole	2.412		
3	Octupole	3.608	3.467	
4	Decapole	5.555	5.381	
5	12-pole	8.753	8.536	8.340
6	14-pole	14.06	13.78	13.53

Maximum Systematic Errors

How big can ΔQ be? No easy answer ...

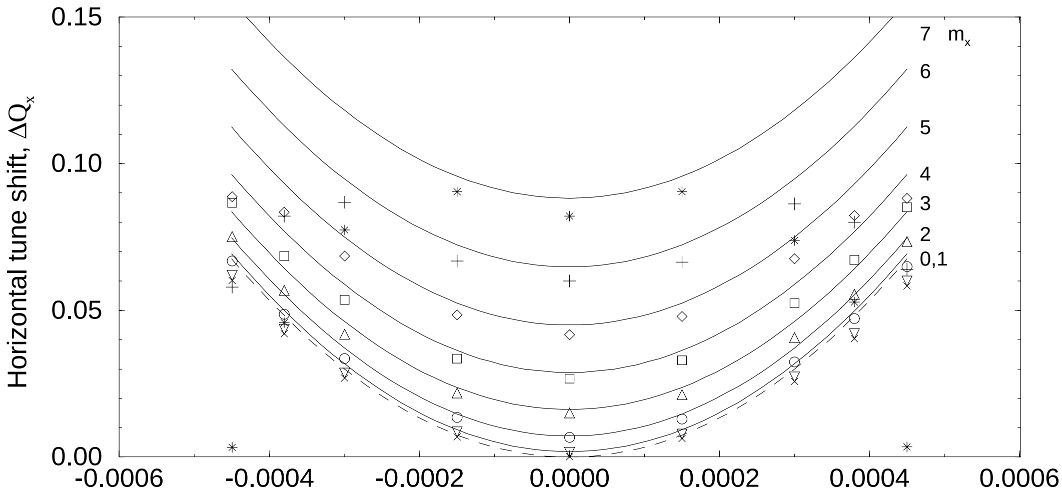
• Beam-beam tune shifts in hadron colliders are $\sim$ .03, with $\sim$ 10 hour lifetimes. Strong resonances.

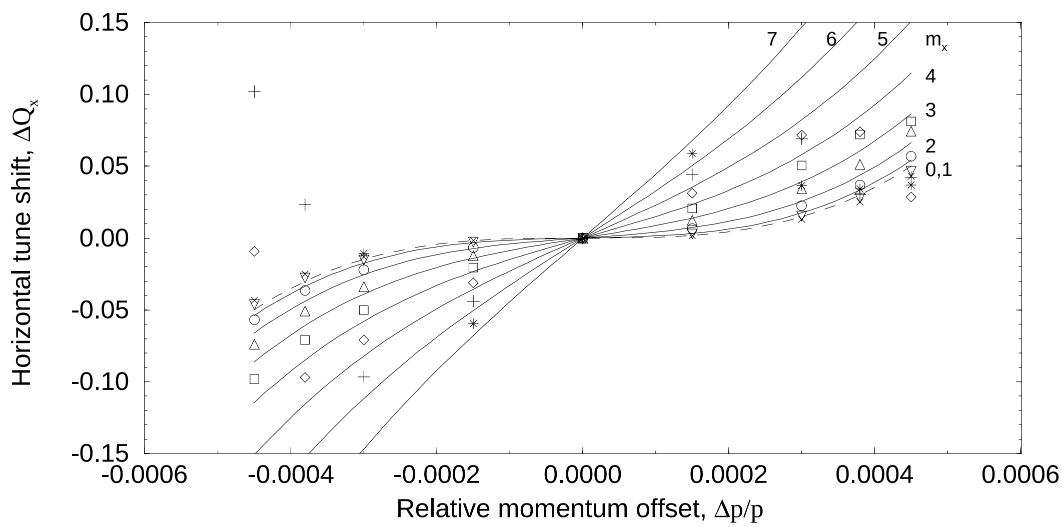
• Space charge tune shifts in boosters are $\sim$ 0.5, but with reduced lifetimes. Weak resonances.

• Try tracking a simple SHORT cell example:

Parameter	units	value
Storage energy	[TeV]	30.0
Injection energy	[TeV]	1.0
Dipole field (store)	[T]	12.5
Transverse RMS emittance, ϵ	[μm]	1.0
Half cell length, L	[m]	110
Max. cell beta, $\widehat{\beta}$	[m]	376
Max. cell dispersion, $\hat{\eta}$	[m]	3.85
Max. betatron size, $\widehat{\sigma}_\beta$	[mm]	.594
Mmtm. width, σ_p/p	[10^{-3}]	.1545

TOP: octupole $b_3 = 5 \times 10^{-4}$ at $r_0 = 16$ mm.





BOTTOM: decapole $b_4 = 30 \times 10^{-4}$.

Lines are prediction, symbols are numerical data.

- **Assumption 3:** the maximum allowable tune shift is $\widehat{\Delta Q}_x \approx 0.1$.

- **Assumption 4:** the extreme tune shift of interest occurs when $m_x = |m_\delta| \equiv m$.

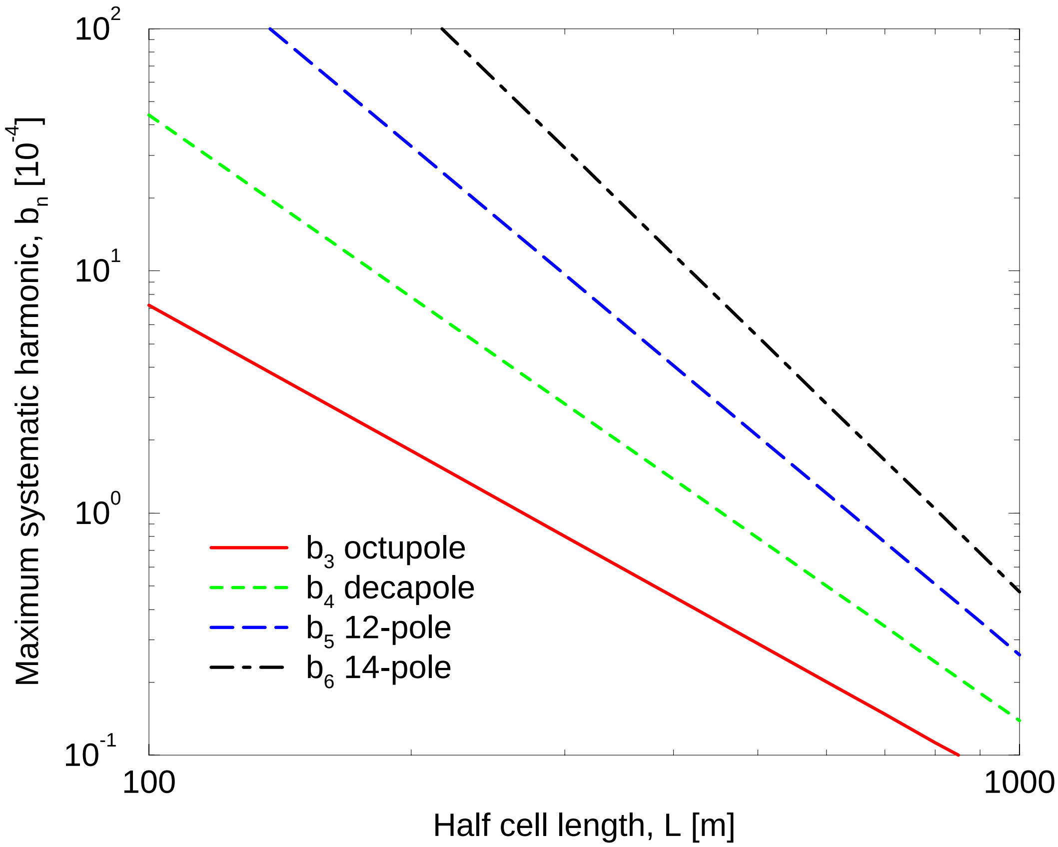
This leads (at last) to the major result for the maximum systematic harmonics

$$\frac{b_n}{r_0^n} \leq \widehat{\Delta Q}_x \frac{1}{D_n} L^{-(n+1)/2} \left(\frac{\beta\gamma}{m^2\,\epsilon_x}\right)^{(n-1)/2} \tag{6}$$

- If $\phi_c = 90$ degrees, the first few values of $D_n(\phi_c)$ are

n	Multipole	D_n
1	Quadrupole	.8333
2	Sextupole	2.412
3	Octupole	6.712
4	Decapole	19.18
5	12-pole	56.49
6	14-pole	170.9

How big can L be? How big can b_n be?



Maximum allowable systematic harmonics versus half cell length, when $\widehat{\Delta Q}_x = 0.1$, $\phi_c = 90$ degrees, $\epsilon_x = 1\mu$m, and $m = 3$, at an energy of 1 TeV, with a reference radius of $r_0 = 16$ mm.

Summary and Conclusions

1. Systematic field errors dominate random field errors in contemporary low temperature superconducting magnets.
2. Presumably systematics will also dominate in HTS magnets. This greatly simplifies the AP analysis.
3. Calculations of tune shifts may be manipulated to give scaling rules for maximum allowable systematics.
4. Lattices with relatively long arc cells have cost saving advantages.
5. Arc cell length may be traded off against systematic field quality.
6. Accelerator physicists are challenged to increase acceptable tune shifts towards 0.1 . Is beam based nonlinear correction necessary?